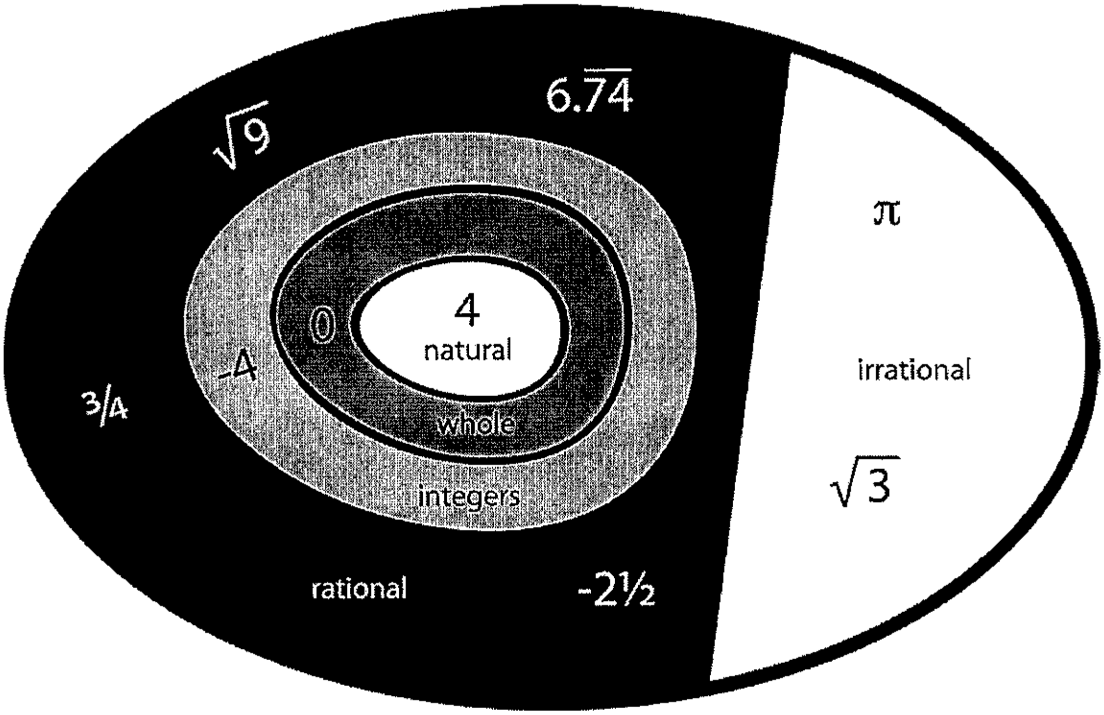
Real Numbers



Natural Numbers: The counting numbers: {1, 2, 3 ...}

Whole Numbers: The set of counting numbers plus zero: {0, 1, 2, 3 ...}

Integers: The set of natural numbers and their opposites plus zero:

{...−3, −2, −1, 0, 1, 2, 3 ...} The set of integers does not include decimals or fractions.

Rational Numbers: Numbers that can be expressed as the ratio of two integers.

Decimal representations of rational numbers either terminate or repeat.

Examples: 2.375, 4, −0.25, $-0.\overline{14}$

Irrational Numbers: Numbers that cannot be expressed as a ratio of two integers.

Their decimal representations neither terminate nor repeat.

Examples: π, $\sqrt{3}$, 0.14114111411114...

Real Numbers: The set of rational and irrational numbers

Real Number System

1. How are the natural and whole numbers different?

2. How are the integers and rational numbers different?

3. How are the integers and rational numbers the same?

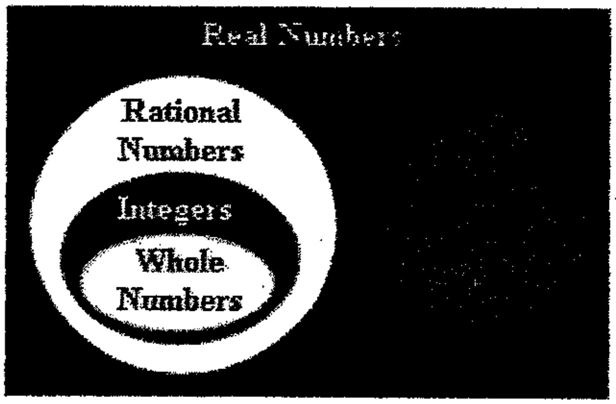
4. How are integers and whole numbers the same?

5. Can a number be both rational and irrational? Use the diagram to explain your answer.

Answer **True** or **False** to the statements below. If the statement is False, explain why.

6. -5 is a rational number.	6. ________________________
7. 0 is an integer.	7. ________________________
8. $\sqrt{16}$ is a natural number	8. ________________________
9. $-3.\overline{25}$ is an integer	9. ________________________
10. $\sqrt{8}$ is rational	10. ________________________
11. $\sqrt{7}$ is a Real number	11. ________________________
12. 18 is a whole number	12. ________________________
13. $-\frac{2}{3}$ is an integer	13. ________________________
14. 2.434434443… is a rational number	14. ________________________
15. 6.57 is an integer	15. ________________________
16. $5.\overline{7}$ is rational.	16. ________________________
17. All fractions are rational numbers.	17. ________________________
18. All integers are whole numbers.	18. ________________________
19. All irrational numbers are Real numbers.	19. ________________________
20. All negative numbers are integers.	20. ________________________

Both rational and irrational numbers are real numbers.



This Venn Diagram shows the relationships between sets of numbers. Notice that rational and irrational numbers are contained in the large blue rectangle representing the set of Real Numbers.

A **rational number** is a number that can be expressed as a fraction or ratio. The numerator and the denominator of the fraction are both integers.

When the fraction is divided out, it becomes a terminating or repeating decimal. (The repeating decimal portion may be one number or a billion numbers.)

Rational numbers can be ordered on a number line.

Examples of rational numbers are :

6 or $\frac{6}{1}$	can also be written as	**6.0**
-2 or $\frac{-2}{1}$	can also be written as	**-2.0**
$\frac{1}{2}$	can also be written as	**0.5**
$\frac{-5}{4}$	can also be written as	**-1.25**
$\frac{2}{3}$	can also be written as	**0.666666666...** $0.\overline{6}$
$\frac{21}{55}$	can also be written as	**0.38181818...** $0.3\overline{18}$
$\frac{53}{83}$	can also be written as	**0.62855421687...** the decimals will repeat after 41 digits

Be careful when using your calculator to determine if a decimal number is irrational. The calculator may not be displaying enough digits to show you the repeating decimals, as was seen in the last example above.

Hint: When checking to see which fraction is larger, change the fractions to decimals by dividing and compare their decimal values.

Examples:

	Which of the given numbers is greater?	Using full calculator display to compare the numbers.
1.	$\frac{2}{3}, \frac{1}{4}$	$.6666666667 > .25$
2.	$-\frac{7}{3}, -\frac{11}{3}$	$-2.333333333 > -3.666666667$

An **irrational number** cannot be expressed as a fraction.
Irrational numbers cannot be represented as terminating or repeating decimals.
Irrational numbers are non-terminating, non-repeating decimals.

Examples of irrational numbers are:

$\pi = 3.141592654.....$
$\sqrt{2} = 1.414213562.....$
and $0.12122122212...$

Note: Many students think that π is the terminating decimal, 3.14, but it is not. Yes, certain math problems ask you to use π as 3.14, but that problem is rounding the value of π to make your calculations easier. π is actually a non-ending decimal and is an irrational number.

Name: ____________________________ Date: ______________

Mrs. Galluzzo

Rational/Irrational #'s

Determine which numbers are rational and which ones are irrational. Place an "I" on the space provided for the irrational numbers and place an "R" on the space provided for the rational numbers.

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________
13. ________
14. ________
15. ________
16. ________
17. ________
18. ________
19. ________
20. ________

Which are rational and which are irrational numbers?

1.	0.424242…	2.	0.424424442…	3.	-0.5638	4.	$0.68\overline{68}$
5.	-.31323334…	6.	$0.383\overline{83}$	7.	0.123123123..	8.	-0.00009
9.	$\frac{4}{5}$	10.	$7.\overline{1234}$	11.	$\frac{34}{3454}$	12.	0
13.	456,812	14.	2.121221222…	15.	34.565656…	16.	3.1416
17.	-0.43854	18.	$\frac{22}{7}$	19.	$0.682\overline{682}$	20.	0.2021223.

Which number below is irrational? Explain.

21. $\sqrt{49}$ 22. $\sqrt{85}$ 23. $\sqrt{\frac{4}{81}}$ 24. $\sqrt{1325}$

Name Date Period

Chapter 11 Workbook Activity 103

Irrational Numbers as Decimals

EXAMPLE Find each root. Tell whether it is *rational* or *irrational*.

$\sqrt{3}$

Using a calculator: $\sqrt{3} = 1.73205...$

The number is *irrational* because it neither ends in zeroes nor has a repeating pattern.

$\sqrt[3]{27}$

Using a calculator: $\sqrt[3]{27} = 3.0$

The number is *rational* because it ends in zeroes.

Directions Complete the chart. Find each root and tell whether it is *rational* or *irrational*. You may use a calculator.

Radical	Root	Rational or Irrational?
$\sqrt{49}$	1) ________	2) ________
$\sqrt{15}$	3) ________	4) ________
$\sqrt{11}$	5) ________	6) ________
$\sqrt{6}$	7) ________	8) ________
$\sqrt{144}$	9) ________	10) ________
$\sqrt{121}$	11) ________	12) ________
$\sqrt{50}$	13) ________	14) ________
$\sqrt[3]{50}$	15) ________	16) ________
$\sqrt{36}$	17) ________	18) ________
$\sqrt[3]{18}$	19) ________	20) ________
$\sqrt{169}$	21) ________	22) ________
$\sqrt[3]{125}$	23) ________	24) ________

Directions Solve the problem.

25) Caitlin has cut out a square piece of graph paper that contains a total of 81 blocks. How many blocks are there along one side of the square?

____

Identifying Rational and Irrational Numbers Name: Homework 9/17/1

Determine if the number is rational (R) or irrational (I).

1) 61π

2) 42

3) $75.082\overline{106}$

4) $\sqrt{101}$

5) $65.4\overline{279}$

6) $^{20}/_{6}$

7) π

8) $5.62\overline{13}$

9) $^{98}/_{16}$

10) 39

11) 89.396668...

12) $\sqrt{17}$

13) 67.714813...

14) $\sqrt{64}$

15) $^{1}/_{4}$

16) $\sqrt{25}$

17) $71.5\overline{186}$

18) $^{7}/_{54}$

19) 20.455566...

20) 97.33997

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______
13. ______
14. ______
15. ______
16. ______
17. ______
18. ______
19. ______
20. ______

1-10	95	90	85	80	75	70	65	60	55	50
11-20	45	40	35	30	25	20	15	10	5	0

2-7

Study Guide and Intervention *(continued)*

Square Roots and Real Numbers

Classify and Order Numbers Numbers such as $\sqrt{2}$ and $\sqrt{3}$ are not perfect squares. Notice what happens when you find these square roots with your calculator. The numbers continue indefinitely without any pattern of repeating digits. Numbers that cannot be written as a terminating or repeating decimal are called **irrational numbers**. The set of **real numbers** consists of the set of irrational numbers and the set of rational numbers together. The chart below illustrates the various kinds of real numbers.

Natural Numbers	{1, 2, 3, 4, ...}
Whole Numbers	{0, 1, 2, 3, 4, ...}
Integers	{..., −3, −2, −1, 0, 1, 2, 3, ...}
Rational Numbers	{all numbers that can be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$}
Irrational Numbers	{all numbers that cannot be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$}

Example **Name the set or sets of numbers to which each real number belongs.**

a. $\frac{4}{11}$ Because 4 and 11 are integers, this number is a rational number.

b. $\sqrt{81}$ Because $\sqrt{81} = 9$, this number is a natural number, a whole number, an integer, and a rational number.

c. $\sqrt{32}$ Because $\sqrt{32} = 5.656854249...$, which is not a repeating or terminating decimal, this number is irrational.

Exercises

Name the set or sets of numbers to which each real number belongs.

1. $\frac{84}{12}$

2. $-\frac{6}{7}$

3. $\frac{2}{3}$

4. $\sqrt{54}$

5. 3.145

6. $\sqrt{25}$

7. 0.62626262...

8. $\sqrt{22.51}$

Write each set of numbers in order from least to greatest.

9. $-\frac{3}{4}, -5, \sqrt{25}, \frac{7}{4}$

10. $\sqrt{0.09}, -0.3131..., \frac{3}{5}$

11. $-1.\overline{25}, 0.05, -\frac{1}{4}, \sqrt{5}$

12. $\frac{5}{4}, -2, \sqrt{124}, -3.11$

13. $-\sqrt{1.44}, -0.35\ \frac{1}{5}$

14. $0.\overline{35}, 2\frac{1}{3}, -\frac{9}{5}, \sqrt{5}$